Multi-Criteria Dimensionality Reduction with Applications to Fairness

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JOINT WORK WITH
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PCA can be unfair!

Standard PCA on face data LFW of male and female

Average reconstruction error (RE) of PCA on LFW

![Graph showing reconstruction error vs. number of features for male and female PCA]
PCA can be unfair!

Standard PCA on face data LFW of male and female

Average reconstruction error (RE) of PCA on LFW

Equalizing male and female weight before PCA

Average reconstruction error (RE) of PCA on LFW (resampled)
Contribution 1: Problem Formulation

*Multi-criteria dimensionality reduction (MCDR):*

\[
\max_{\text{projection } P} \ g(f_1(P), f_2(P), \ldots, f_k(P))
\]

Utility criterion \(f_i\)'s and social welfare \(g\)
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\max_{\text{projection } P} g(f_1(P), f_2(P), \ldots, f_k(P))
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Utility criterion \(f_i\)'s and social welfare \(g\)

- **Mar-Loss**: \(\min_P \max_{i \in \{1, \ldots, k\}} \left( \max_Q \|A_i Q\|_F^2 - \|A_i P\|_F^2 \right)\)

- **NSW**: \(\max_P \prod_{i=1}^k \|A_i P\|_F^2\)
Contribution 2: Algorithms and Guarantees

On linear $f_i$ in $PP^T$ and concave $g$:

• Polynomial-time algorithm for MCDR with optimal utility and small rank violation $s = \sqrt{2k + 1/4} - 3/2$

• Approximation ratio $1 - s/d$ on utility when no rank violation
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• Semi-definite Program (SDP) $\rightarrow$ Multiplicative Weight (MW) method
  • scalable up to $\approx 1000$ dimensions
Contribution 2: Algorithms and Guarantees

- Mar-Loss:

$$\min_P \max_{i \in \{1, \ldots, k\}} \left( \max_Q \|A_iQ\|_F^2 - \|A_iP\|_F^2 \right)$$

- NSW:

$$\max_P \prod_{i=1}^k \|A_iP\|_F^2$$
Contribution 3: Optimization Theory

- Every extreme point of the semi-definite program relaxation of MCDR has low rank
  - Generalize work on low-rank property in semi-definite program by Barvinok’95, Pataki’98
- Optimization result + ML application
Contribution 4: Complexity of MCDR

• NP-hard for general $k$
  • Reduction to MAX-CUT

• Polynomial-time for fixed $k$
  • Algorithmic theory of quadratic maps.
More details

• Poster: Thursday Dec 12\textsuperscript{th} at 10:45 AM -- 12:45 PM, East Exhibition Hall B + C #80
• Happy to chat!

• Code: github.com/uthaipon/multi-criteria-dimensionality-reduction
• Web: sites.google.com/site/ssamadi/fair-pca-homepage

(searchable links at NeurIPS website or my website Uthaipon Tantipongpipat)